[9.7] Derive the expression



1

1

**ℜ**

**I**

1. 

* Start with standard unit circle
*  is equation of circle of radius r
* Thus  is equation of unit circle
* Center: Origin = (0, 0)
* Starting point: *z* = 1 on real axis
* Trajectory is Counter-clockwise (CCW)

as  goes from 0 to 2

1

1

**ℜ**

**I**

2. 

* Reverse the direction
* Center: Origin = (0,0)
* Starting point: z = 1
* Clockwise (CW) trajectory

l / 2

**ℜ**

**I**

l / 2

3. 

* Expand radius
* Multiply by 
* Starting point: z = 

l / 2

l / 2

**ℜ**

**I**

4. 

* Rotate circle CCW by 90°
* New starting point at z = l i / 2 on

Imaginary axis

*  is equation of unit circle

and *i* is at 90° on unit circle ⇒ 

* Equation is multiplied by i

*  shows points on circle are rotated CCW by 90°

**ℜ**

l / 2

**I**

- l / 2

5. 

* Translate circle downward by l / 2
* Simply subtract :



* New center: z = 
* New starting point: Origin
* Trajectory: CW